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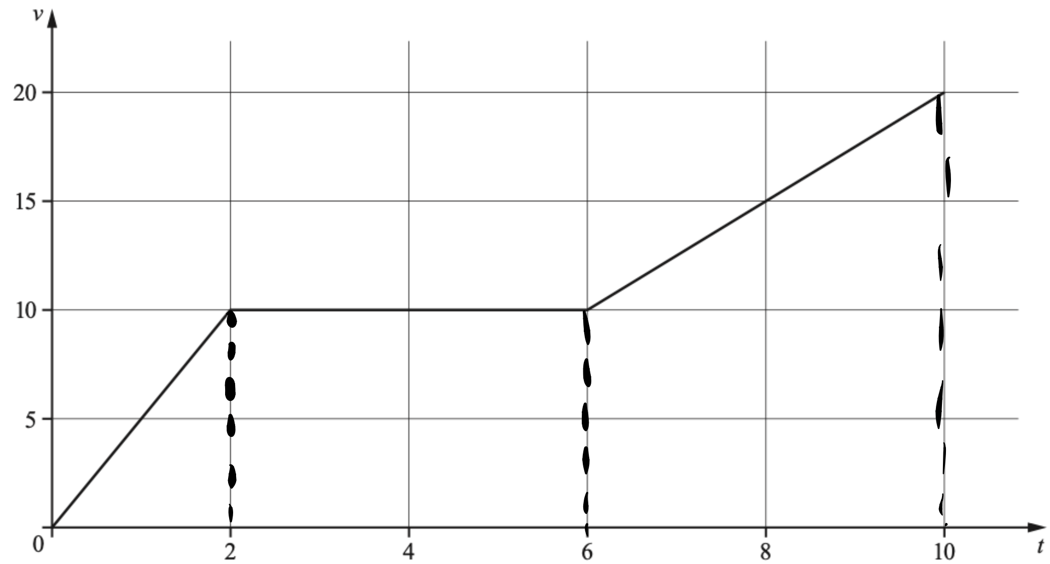
1. (a)



The diagram shows the velocity-time graph of a particle *P* moving in a straight line with velocity $v\ ms^{-1}$ at time *t* seconds after leaving a fixed point.

I. Write down the value of the acceleration of *P* when *t* = 5.

0

[1]

II. Find the distance travelled by the particle *P* between *t* = 0 and *t* = 10.

[2]

$A = \frac{1}{2} \times 2 \times 10 = 10$

$B = 4 \times 10 = 40$

$C = \frac{1}{2}(30) \times 4 = 60$

110

(b) A particle Q moves such that its velocity, $v\,ms^{-1}$, t seconds after leaving a fixed point, is given by $v = 3\sin 2t - 1$.

I. Find the speed of Q when $t = \frac{7\pi}{12}$.

$v = 3\sin\frac{7\pi}{6} - 1$ [2]

$= -\frac{5}{2}$

$S = \frac{5}{2}\,ms^{-1}$

II. Find the least value of t for which the acceleration of Q is zero.

$a = \frac{dv}{dt} = 6\cos 2t$ [3]

$6\cos 2t = 0$

$2t = \cos^{-1}(0)$

$2t = \frac{\pi}{2}$

$t = \frac{\pi}{4}$

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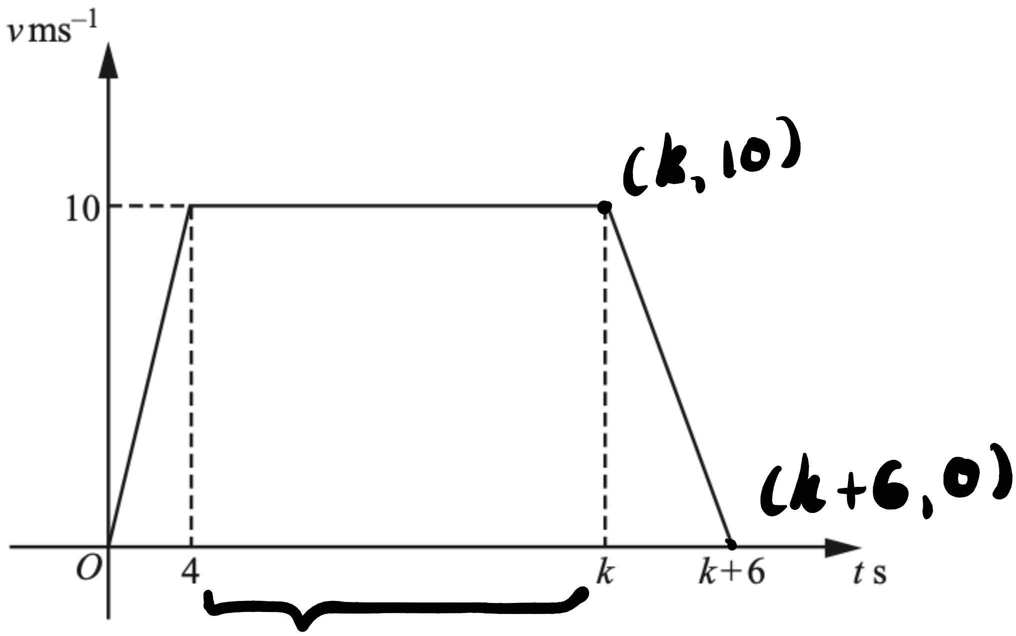
2.



The velocity-time graph represents the motion of a particle travelling in a straight line.

a. Find the acceleration during the last 6 seconds of the motion. [1]

$$\frac{\text{rise}}{\text{run}} = \frac{-10}{6} = -\frac{5}{3}$$

b. The particle travels with constant velocity for 23 seconds. Find the value of *k*. [1]

$$k-4 = 23$$
$$k = 27\text{s}$$

c. Using your answer to **part (ii)**, find the total distance travelled by the particle. [3]

$$\frac{1}{2}(33+23) \times 10$$
$$\frac{56}{2} \times 10 = 280\text{m}$$

3. The velocity, $v\,ms^{-1}$, of a particle travelling in a straight line, t seconds after passing through a fixed point O, is given by $v = \frac{4}{(t+1)^3}$.

a. Explain why the direction of motion of the particle never changes. [1]

$v = 0$ $\quad 4 \neq 0$

$\frac{4}{(t+1)^3} = 0$ $\quad \therefore$ direction never changes

b. Showing all your working, find the acceleration of the particle when $t = 5$. [3]

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(4 \times (t+1)^{-3}\right)$$

$$= -12(t+1)^{-4}$$

$$= -12(5+1)^{-4} = \frac{-12}{6^4} = -\frac{1}{108}$$

c. Find an expression for the displacement of the particle from O after t seconds. [3]

$$s = \int v\,dt$$

$$= \int 4(t+1)^{-3}\,dt$$

$$= \frac{4}{-2}(t+1)^{-2} + C$$

$$= -2(t+1)^{-2} + C$$

$s = 0,\ t = 0$

$0 = -2(1)^{-2} + C$

$C = 2$

$\therefore s = -2(t+1)^{-2} + 2$

d. Find the distance travelled by the particle in the fourth second. [2]

$$t = 4,\ s = -2(5)^{-2} + 2 = \frac{-2}{25} + 2 = \frac{48}{25}$$

$$t = 3,\ s = -2(4)^{-2} + 2 = \frac{-2}{16} + 2 = \frac{15}{8}$$

$$4^{th}\text{ sec} = 4^{th} - 3^{rd}$$

$$= \frac{48}{25} - \frac{15}{8} = \frac{9}{200}$$

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4. A particle travelling in a straight line passes through a fixed point *O*. The displacement, *x* metres, of the particle, *t* seconds after it passes through *O*, is given by $x = 5t + \sin t$.

a. Show that the particle is never at rest.

$$v = \frac{dx}{dt} = 5 + \cos t$$

[2]

$v = 0$

$\cos t = -5$

$t = \cos^{-1}(-5)$

(no solution)

$\therefore$ particle is never at rest.

b. Find the distance travelled by the particle between $t = \frac{\pi}{3}$ and $t = \frac{\pi}{2}$.

$x = 5t + \sin t$

[2]

$t = \frac{\pi}{3}$, $x = \frac{5\pi}{3} + \sin\frac{\pi}{3}$

$= \frac{5\pi}{3} + \frac{\sqrt{3}}{2} = 6.102$

$t = \frac{\pi}{2}$, $x = \frac{5\pi}{2} + \sin\frac{\pi}{2}$

$= \frac{5\pi}{2} + 1 = 8.854$

Distance $= 2.752$ m

c. Find the acceleration of the particle when $t = 4$.

[2]

$$v = 5 + \cos t$$

$$a = -\sin t$$

$$a = -\sin(4)$$

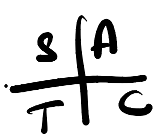
$$a = 0.757 \, ms^{-2}$$

d. Find the value of t when the velocity of the particle is first at its minimum.

[2]

$$v = 5 + \cos t$$

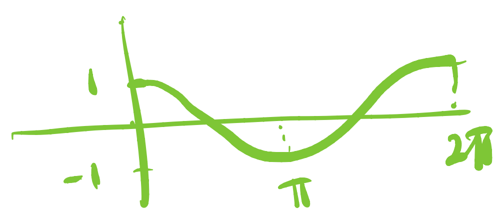
$$\frac{dv}{dt} = -\sin t$$



$$-\sin t = 0$$

$$t = \sin^{-1}(0)$$

$$t = 0, \pi$$



$$t = \pi$$

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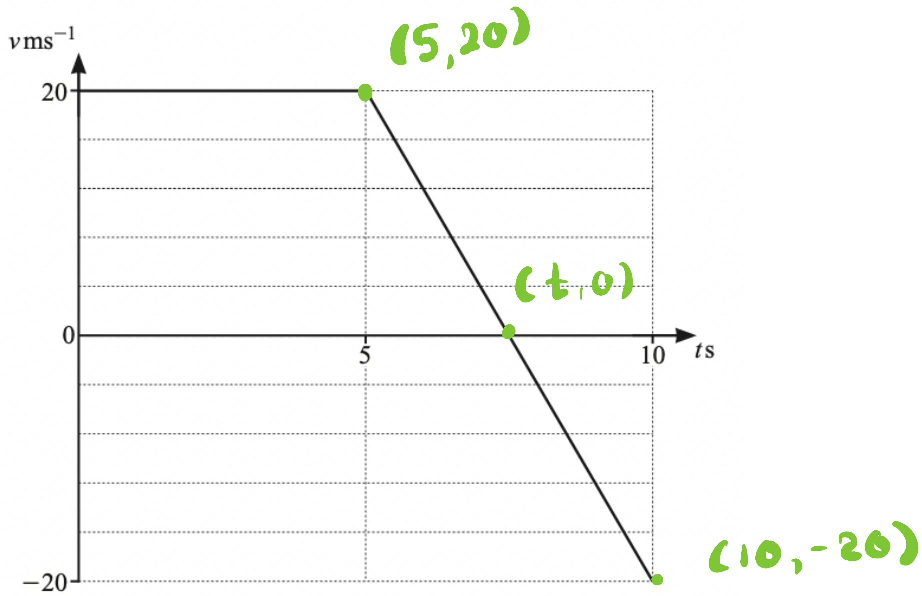
5. (a)



The velocity-time graph for a particle *P* is shown by the two straight lines in the diagram.

(i) Find the deceleration of *P* for $5 \leq t \leq 10$.

[2]

$m = \frac{\text{rise}}{\text{run}} = \frac{-40}{5} = -8\,ms^{-2}$

deceleration $= 8\,ms^{-2}$

(ii) Write down the value of *t* when the speed of *P* is zero.

[1]

$\frac{20}{t-5} = 8$ | $\frac{20}{8} = t-5$, $\frac{5}{2} = t-5$ | $t = 7.5s$

(iii) Find the distance *P* has travelled for $0 \leq t \leq 10$.

[2]

$\frac{1}{2}(10+5) \times 40$

$15 \times 20 = 300\,m$

(b) A particle Q has a displacement of x m from a fixed point O, t s after leaving O. The velocity, $v\ ms^{-1}$, of Q at time ts is given by $v = 6e^{2t} + 1$.

(i) Find an expression for x in terms of t.

[3]

$x = \int v\, dt$

$= \frac{6}{2} e^{2t} + t + c$

$= 3e^{2t} + t + c$

when $t = 0, x = 0$

$0 = 3 + 0 + c$

$c = -3$

$\therefore x = 3e^{2t} + t - 3$

(ii) Find the value of t when the acceleration of Q is 24 ms^{-2}.

$v = 6e^{2t} + 1$

[3]

$a = 12e^{2t}$

$12e^{2t} = 24$

$e^{2t} = 2$

$2t = \ln 2$

$t = \frac{1}{2} \ln 2$

6. A particle is moving in a straight line such that *t* seconds after passing a fixed point *O* its displacement, *s* m, is given by $s = 3sin\,2t + 4cos2t - 4$.

(i) Find expressions for the velocity and acceleration of the particle at time *t*. [3]

$v = 6\cos 2t - 8\sin 2t$

$a = -12\sin 2t - 16\cos 2t$

(ii) Find the first time when the particle is instantaneously at rest. [3]

$v = 0$

$6\cos 2t - 8\sin 2t = 0$

$3\cos 2t - 4\sin 2t = 0$

$3\cos 2t = 4\sin 2t$

$\frac{3}{4} = \tan 2t$

$2t = \tan^{-1}(3/4)$

$t = 0.322\,s$

(iii) Find the acceleration of the particle at the time found in **part (ii)**. [2]

$a = -12\sin 2t - 16\cos 2t$

$t = 0.322$,

$a = -12\sin(2\times 0.322) - 16\cos(2\times 0.322)$

$= -20$